Stability Measure of Postural Dynamic Equilibrium based on Residual Radius

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Abstract. This paper formalizes the robustness of a virtual human dynamic equilibrium through the residual radius of its admissible generalized force set. The admissible generalized force set is defined as the image of the contact force constraints (corresponding to the Coulomb model) in the generalized force space. This set is approximated by a polyhedron and its residual radius is computed using a linear program. The measure relevance is analysed from experimental data of a sit-to-stand motion on which residual radius is evaluated.

Key words: equilibrium robustness, residual radius, human motion, postural stability

Introduction

Generating whole body motion for virtual human that appears natural is a long standing problem in character animation, highligting the lack of a solution to Bernstein's redundancy problem. A lot of studies have identified invariants in various motions such as minimum jerk for reaching movements [1], minimum torque change, minimum muscle tension change, minimum motor command change, minimum of biological noise, etc.). Such models are well suited to generate reference trajectories which can be further modified to compensate expected perturbations in a feedforward way. Additionaly, sensory-driven feedback strategies are needed to cope with unexpected disturbances.

However, some strong perturbations cannot be compensated due to limitations in contact forces and joint torques. Some "distance to constraint violation" may therefore be monitored in order to ensure equilibrium by triggering adjustment motion when necessary.

The "quality" of equilibrium for humanoid robots is generally measured as the distance between some caracteristic point (ZMP, CdM projection, FRI) and the support polygon borders. Popovic [7] offers a comprehensive review of those caracteristic points and study their evolution during human walk. But these measures are only valid if the contacts are coplanar. Harada [3] extended the ZMP measure in order to handle situations where the upper-limbs are in contact with the environment. However, the limits in frictionnal contact forces are still not taken into account.

To overcome this restriction the force closure measures which have been developped in the past for grasp and fixture analysis can be revisited. Wieber [8], then Hirukawa *et al.* [4] have proposed a formulation of the frictional constraints as a feasable contact wrench domain that has lead to a more universal measure, however, to our knowledge, this measure has never been used to evaluate the set of possible human motions.

This paper is concerned by the quantification of a posture quality regarding the dynamical balance. We first consider a single body in contact and show how the set of feasable contact wrenches can be computed (1). We then define its residual radius as a measure of the contact set quality (2). This method is then generalized to the case of a virtual human in contact with its environment 3. At last, the relevance of the measure is evaluated on a sit-to-stand motion.

1 Resistable and Applicable Contact Wrenches

Let us consider a rigid body b in contact at m points with bodies e_i ($i = 1, \ldots, m$) from its environment.

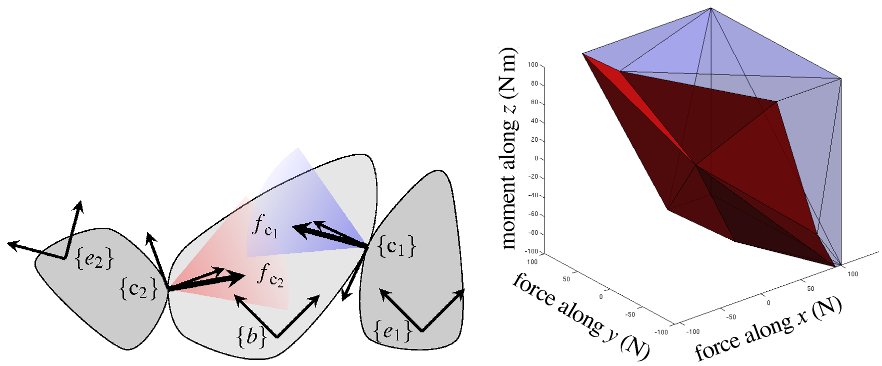

Fig. 1 A rigid body b in contact with two other bodies (left) and the set of resistable contact wrench $w_b \in \mathscr{W}_c$ (right). The wrench is expressed in the frame $\{b\}$.

For each contact point (let us say the i-th), we can define the frame $\{c_i\}$ and the contact force f_{c_i}. Considering the punctual with friction contact model, when no sliding occurs, the overall friction force lies within a revolution cone:

$$\mathscr{F}_{c_i} = \left\{ f_{c_i} : \sqrt{f_{c_i}^{T} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} f_{c_i}} \le \mu_i \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} f_{c_i} \right\} \tag{1}$$

where μ_i is the static coefficient of friction. The set of wrenches that these contact forces can produce is called the set of *resistable contact wrench* $\mathscr{W}_\mathrm{c}$:

$$\mathscr{W}_\mathrm{c} = \left\{ w_\mathrm{c} : w_\mathrm{c} = G f_\mathrm{c} ; f_\mathrm{c} \in \mathscr{F}_{\mathrm{c}_1} \times \cdots \times \mathscr{F}_{\mathrm{c}_m} \right\} \quad \text{with } f_\mathrm{c} = \begin{bmatrix} f_{\mathrm{c}_1} \\ \vdots \\ f_{\mathrm{c}_m} \end{bmatrix} \tag{2}$$

If the body b is a manipulated object, G is called the *grasp matrix*. A planar example is depicted in figure 1, where the resistable contact wrench set is unbounded but does not span the entire $\mathbb{R}^3$.

The image of a revolution cone through a linear application (ie. the grap matrix) cannot be computed directly. Therefore, we approximate it by a linear cone. Let us choose p regularly spaced unitary vectors on the Coulomb cone border of the i-th contact and call them $f_{\mathrm{c}_i}^j$ (where $j = 1..p$). The set spanned by their positive combinations is a convex linear cone:

$$\mathscr{F}_{\mathrm{c}_i}^* = \left\{ f : f = \sum_{j=1}^{p} a_j f_{\mathrm{c}_i}^j ; a_j \geq 0 \right\} = \mathrm{pos}\left(f_{\mathrm{c}_i}^1, \ldots, f_{\mathrm{c}_i}^p \right) \tag{3}$$

which (as any polyhedron) can also be defined as a generalized inequality:

$$\mathscr{F}_{\mathrm{c}_i}^* = \{ f : A_{\mathrm{c}_i} f \leq b_{\mathrm{c}_i} \} \tag{4}$$

where the rows of A_{c_i} are vectors chosen normal to the facets and outgoing. The set $\left\{ f_{\mathrm{c}_i}^1, \ldots, f_{\mathrm{c}_i}^m \right\}$ is called the *vertice representation* of $\mathscr{F}_{\mathrm{c}_i}^*$ and A_{c_i} and b_{c_i} are called its *half-space representation*.

The image $\mathscr{W}_\mathrm{c}^*$ of $\mathscr{F}_{\mathrm{c}_i}^* \times \cdots \times \mathscr{F}_{\mathrm{c}_m}^*$ through the grasp map is also given by a positive combination:

$$\mathscr{W}_\mathrm{c}^* = \left\{ w_\mathrm{c} : w_\mathrm{c} = \sum_{i=1}^{m} \left(S\, {}^{\mathrm{c}_i}\!Ad_\mathrm{b} \right)^\mathrm{T} f_{\mathrm{c}_i} \text{ where } f_{\mathrm{c}_i} \in \mathscr{F}_{\mathrm{c}_i}^* \right\} \tag{5}$$

$$= \mathrm{pos}\left(w_{\mathrm{c}_1}^1, \ldots, w_{\mathrm{c}_i}^j, \ldots, w_{\mathrm{c}_m}^p \right) \text{ where } w_{\mathrm{c}_i}^j = \left(S\, {}^{\mathrm{c}_i}\!Ad_\mathrm{b} \right)^\mathrm{T} f_{\mathrm{c}_i}^j \tag{6}$$

When a grasping problem is considered, the contact forces are often actively controlled and also limited by additionnal (actuators-related) constraints. $\mathscr{W}_\mathrm{c}$ is then replaced by the set of *applicable contact wrench* $\overline{\mathscr{W}}_\mathrm{c}$:

$$\overline{\mathscr{W}}_\mathrm{c} = \left\{ w_\mathrm{c} : w_\mathrm{c} = G f_\mathrm{c} ; f_\mathrm{c} \in \mathscr{F}_{\mathrm{c}_1} \times \cdots \times \mathscr{F}_{\mathrm{c}_m} : \chi\left(f_\mathrm{c} \right) = 1 \right\} \tag{7}$$

$$\text{where } \chi\left(f_\mathrm{c} \right) = \begin{cases} 1 \text{ if the additionnal constraints hold} \\ 0 \text{ otherwise} \end{cases} \tag{8}$$

A common choice for χ is to limit the total normal contact force, because its approximation leads to the following convex hull:

$$\overline{\mathscr{W}}_{\mathrm{c}}^{*} = \alpha \left\{ w : w = \sum_{i=1}^{m} \sum_{j=1}^{p} a_{i,j} w_{\mathrm{c}_i}^{j} ; 0 \leq a_{i,j} \leq 1 \right\} = \alpha \ \mathrm{conv}\left(0, w_{\mathrm{c}_1}^{1}, \ldots, w_{\mathrm{c}_i}^{j}, \ldots, w_{\mathrm{c}_m}^{p}\right) \tag{9}$$

where $\alpha \in \mathbb{R}^{+}$ is a scale factor. Mishra [6] proposed several other choices for χ.

2 Residual Ball and Radius

2.1 Definition

For a manipulation task, $\overline{\mathscr{W}}_{\mathrm{c}}$ (or $\mathscr{W}_{\mathrm{c}}$) is strongly related to the grasp quality: it can be used to check whether some expected external wrench w_e (for instance the wrench of gravity and inertial effects) is sustainable or not (simply check if $w_e \in \overline{\mathscr{W}}_{\mathrm{c}}$). Moreover, the "further" w_e is from the boundary of $\overline{\mathscr{W}}_{\mathrm{c}}$, the "more robust" is the grasp. Kirkpatrick et al. [5] used the *residual radius* of $\overline{\mathscr{W}}_{\mathrm{c}}$ (or $\mathscr{W}_{\mathrm{c}}$) as a quantitative measure of this robustness.

Let us consider the largest hypersphere centered at w_e and fully contained inside $\overline{\mathscr{W}}_{\mathrm{c}}$. The hypersphere is called the *residual ball* and its radius the *residual radius*. Physically, the residual radius is the norm of the largest wrench which can be sustained in any direction.

2.2 Implementation

Finding the largest hypersphere included in the polyhedron $\mathscr{W}_{\mathrm{c}}^{*}$ can be written as the following linear program (LP): maximize r subject to $dr \leq b - Ac$
where A, b are $\mathscr{W}_{\mathrm{c}}^{*}$ half-space representation, c is the hypersphere center and d is computed from A as follows: $A = (a_{ij}) \qquad d = (d_i) \qquad d_i = \sum_j \sqrt{a_{ij}^2}$

In what follows, we make use of the polyhedral computation library *cddlib* written by Fukuda [2] to compute the half-space representations (A and b).

The process of computing the residual radius can be summarized as follows:

1. for each contact, compute the vertices $f_{\mathrm{c}_i}^{j}$,
2. compute each vertice image in the wrench space $w_{\mathrm{c}_i}^{j}$,
3. compute the half-space representation of $\mathscr{W}_{\mathrm{C}}^{*}$,
4. compute the center c of the sphere (which is given by the external wrenches),
5. solve the LP to compute the residual radius.

3 Virtual Human Dynamics

Let us consider the problem of controlling a multi-legged system such as a human. These systems are underactuated and thus rely on the contact forces to produce motion. Therefore, the limitations holding on the contact forces reduce the range of possible motions. We will show here how these limitations can be accounted for in the configuration space and then generalize the residual radius measure.

3.1 Equations of Motion

We model the virtual human as set of rigid bodies linked together by a tree-like structure. The mechanism has $n = 36$ degrees of freedom (dof). Let us denote $q \in \mathbb{R}^n$ the generalized coordinates of these joints. The whole system can be viewed as a free flying robot, whose configuration is given by the vector q and the pose of a "reference" body H_r (see figure 2). The generalized velocities and accelerations are then respectively $\nu = \begin{bmatrix} \nu_r \\ \dot{q} \end{bmatrix}$ and $\dot{\nu} = \begin{bmatrix} \dot{\nu}_r \\ \ddot{q} \end{bmatrix}$.

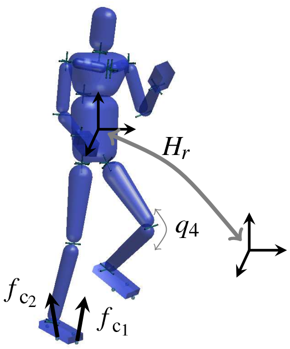

Fig. 2 The virtual human with 36 active dof and the homogeneous transformation matrix accounting for the position of the free floating reference body (the pelvis).

Assuming that all the contacts occur with static bodies, the condition of adhesion and non-lifting are written as $3m$ kinematic constaints:

$$0 = J_c(q)\dot{\nu} + \dot{J}_c(q,\dot{q})\nu \tag{10}$$

where $J_c \in \mathbb{R}^{(3m\times(n+6))}$ is the jacobian of all the contact points.

The Newton-Euler equations of motion are given by

$$M(q)\dot{\nu} + N(q,\nu)\nu = g(H_r,q) + S\gamma(t) + J_c(q)^T f_c \tag{11}$$

where $M,N \in \mathbb{R}^{(n+6)\times(n+6)}$ are respectively the inertia and non-linear effects matrices, $g \in \mathbb{R}^{n+6}$ is the vector of gravitational generalized forces, and $\gamma(t) \in \mathbb{R}^n$ are the control input functions which are mapped with the constant matrix $S = \begin{bmatrix} 0_{6\times6} \\ I_{n\times n} \end{bmatrix}$ into the actively controlled joints. Eventually, f_c accounts for the contact forces, which can also be viewed as the Lagrangian multipliers of the contact-related kinematic constraints.

3.2 Resistable Generalized Contact Force

When considering the virtual human equilibrium, we are interested in the impact of the contact force limitations on the set of admissible generalized forces. In the section 1, we mapped the Coulomb cones through the grasp matrix into the resistable contact wrench, we may extend this method by mapping the friction cone through the transposed contact point jacobian into the generalized force space $\mathscr{T}_{\mathrm{c}}$. Its linear approximation $\mathscr{F}^*_{\mathrm{c}_m}$ is given by:

$$\mathscr{T}^*_{\mathrm{c}} = \left\{ \tau : \tau = J_{\mathrm{c}}(q)^{\mathrm{T}} f_{\mathrm{c}} : f_{\mathrm{c}} \in \mathscr{F}^*_{\mathrm{c}_i} \times \ldots \times \mathscr{F}^*_{\mathrm{c}_m} \right\} \tag{12}$$

$$= \mathrm{pos}\left(\tau^1_{\mathrm{c}_1}, \ldots, \tau^j_{\mathrm{c}_i}, \ldots, \tau^p_{\mathrm{c}_m} \right) \text{ where } \tau^j_{\mathrm{c}_i} = J_{\mathrm{c}_i}(q)^{\mathrm{T}} f^j_{\mathrm{c}_i} \tag{13}$$

$\mathscr{T}^*_{\mathrm{c}}$ is a polyhedron into $\mathbb{R}^{(n+6)}$ which represents the set of generalized contact forces which can be sustained by the contact forces. One can compute its residual radius as in section 2.2. However, it is harder to give a physical meaning to the radius in this space. Moreover, computing the half-space representation of this polyhedron in this high-dimensionnal space is time consuming. Therefore, we will only consider a subset of the generalized forces. As the ground force limitations are much more restrictive for the unactuaded DOFs than for the actuated ones (because in the latter case, the actuators can compensate for the restrictions), we choose to consider the generalized forces corresponding to the unactuated DOFs. In the case of a virtual human, the only unactuated DOFs are the ones positionning the root body (the bust), therefore the 6 corresponding generalized forces consist of a wrench and the measure developped for the grasping can be applied directly by replacing G with the 6 first lines of $J_{\mathrm{c}}(q)^{\mathrm{T}}$.

Let us multiply the dynamic equilibrium equation with $P^{\perp} = \left[I_{6\times6} \; 0_{36\times36} \right]$, it comes:

$$P^{\perp} \left(M(q)\dot{v} + N(q,v)v - g(H_{\mathrm{r}}, q) \right) = P^{\perp} J_{\mathrm{c}}(q)^{\mathrm{T}} f_{\mathrm{c}} \tag{14}$$

One can then compute the residual radius measure on the resulting 6 dimensionnal space as follow:

1. for each contact, compute the vertices $f^j_{\mathrm{c}_i}$,
2. compute each vertice image through $P^{\perp} J^{\mathrm{T}}_{\mathrm{c}}$,
3. compute the half-space representation of the polyhedron,
4. compute the center $c = P^{\perp} \left(M(q)\dot{v} + N(q,v)v - g(H_{\mathrm{r}}, q) \right)$, of the sphere,
5. solve the LP to compute the residual radius.

3.3 Evaluation on Human Motion

In order to evaluate the relevance of this measure, we computed it on a sit-to-stand motion recorded with an optical motion tracking system.

The subjet is initially sitting on a stool, with its bottom in contact with the stool and each foot in contact with the ground. We modeled the contact between each foot and the ground as well as between each thick and the stool with respectively 4 and 1 punctual with friction contacts (see figure 3).

We computed the measure as detailed in the section 3.2, using the captured motion as input. We choose the frame $\{b\}$ to be parallel to the inertial frame with

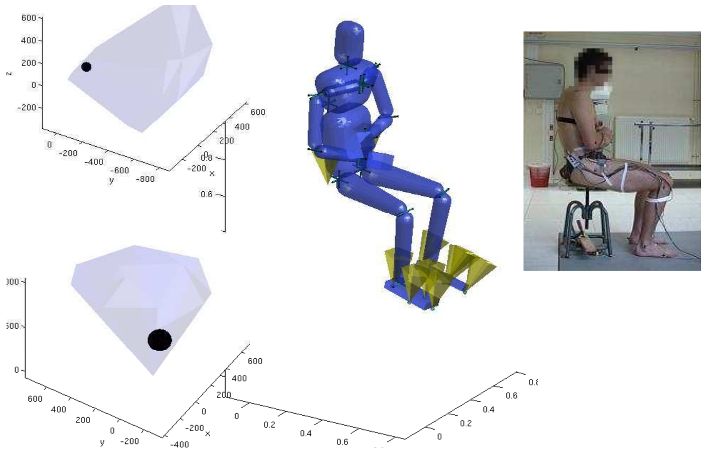

Fig. 3 (*right*) The virtual human at the beginning of the sit-to-stand motion, in contact with the ground and the seat. The linearized friction cones appear in yellow. (*top left*) Resistable contact moment (numerical values in Nm). (*bottom left*) Resistable contact force (numerical values in N).

its origin at the virtual human center of mass. The corresponding set of resistable generalized contact force space at the beginning of the motion is showed figure 3 together with the residual ball.

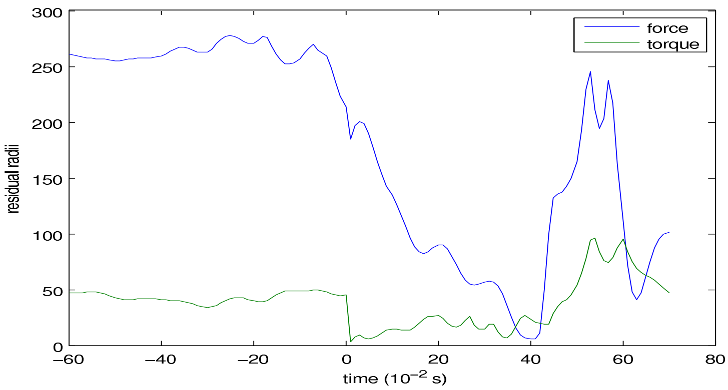

Fig. 4 Residual radii of the contact wrench (force and torque) evaluated over the time during a sit-to-stand motion. The origin of time is chosen at the lift-off.

Figure 4 shows the evolution of the residual radii of both torque and force components during the motion. One can notice that:

- there is a discontinuity when the seat contacts lift off,
- the radii present a maximum when the normal force is the highest, that is when the vertical acceleration is maximal ;

- the radii reach low level, indicating that the system is really near to tip.

This evolution is very conform to expectations and suggests that the proposed measure can be successfully used to monitor the equilibrium stablility during a motion.

Conclusion and perspectives

We defined a measure of dynamic equilibrium robustness and showed its relevance on a sit-to-stand motion. Further work is needed to study how sensitive the measure is to errors in contact parameters estimation. Checking the measure behaviour against other human motions, typically involving contacts between the upper limbs and the environment would also be usefull. Eventually, we could use the measure to adjust a virtual human posture in order to improve its equilibrium robustness.

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