Dynamic sliding mode control of a four-wheel skid-steering vehicle in presence of sliding

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Abstract The paper deals with the design and the implementation of a robust path-following feedback controller, based on the dynamic model of a four-wheel skid-steering robot, performing high speed turns. The control inputs are respectively the linear velocity and the yaw angle. The main object of this paper is to elaborate a sliding mode controller, proved to be robust enough to avoid the knowledge of the forces within the wheel-soil interaction, in the presence of sliding phenomena. Finally, a 3D simulation is performed with an accurate physical engine to evaluate the efficiency of this designed control law.

1 Introduction

Vehicle systems are not usually easy to control owing to unknowns about their model and owing to the difficulty to evaluate the forces in the wheel-soil interaction. Many interaction models developped by Bakker et al. (1987) or by Pacejka (2002) try to represent the complexity of the physic phenomenon. However, wheel-soil interaction is still one of the great unknowns in mobil robotic systems.

The dynamic of this class of systems has been studied by Caracciolo et al. (1999), with the use of a dynamic feedback linearization paradigm for a model-based controller that minimizes lateral skidding by imposing the longitudinal position of the instantaneous center of rotation. Now, we offer an other strategy without nonholonomic contraint, by using a sliding mode control law.

This kind of controller, developped by Utkin (1992), authorizes a decoupling design procedure, disturbance rejection, insensitivity to dynamic parameters variations, and a simple implementation. That is why this control law has been treated in many ways in the literature. A. et al. (1994) and Aguilar et al. (1997) studied dynamic control laws, but without taking into account the complex dynamical model of the vehicle. Yang and Kim (1999) and then Corradini and Orlando (2002) studied the dynamical model of a unicycle for the design of the controller by using a nonholonomic constraint, considering a null lateral velocity. With Hamerlain et al. (2005), it is taken into account that in realistic case, the nonholonomic constraints are not satisfied. But the problem is addressed

Figure 1. RobuROC4

for a partially linearized dynamical model of a unicycle robot.
Here, we suggest an original dynamical model based sliding modes control method for fast autonomous mobile robots, that controls the couples applied in the wheels. The main objective is to follow a given path with a relatively high speed by servoing the longitudinal velocity and the yaw.
The terrains considered here are horizontal and relatively smooth compared to the size of the wheels. If the most part of the mobile robot motion controllers use the hypothesis of rolling without slipping, it is no longer suitable at high speed where wheel slip can not be neglected. Owing to the dynamics of the vehicle and the saturation of admissible forces by the soil, the slippage reduces the robot motion stability. So that we need a controller robust enough.
A 3D simulation is done in a dynamic environment, using an interaction wheel-soil model of forces designed by Szostak et al. (1988), described in section four. We will analyze the motion control of a RobuROC 4 represented in figure 1. It is an electric car designed and manufactured by the Robosoft society which consists of a four driven wheels. Each one of the four wheels is independently actuated. Wheel sensors, a GPS and a a gyro meter are necessary for the control law implementation.
This paper is organized as follows. In the second section, the system dynamical model is given. In the third section, we describe the design of the sliding mode controller. In the last section, simulations results using this controller are presented and analyzed.

2 System Dynamic Model

A dynamic model of a skid-steering vehicle is established in fixed frame $[x; y; \theta]^T$. The representation of the 4WD skid-steering vehicle is described on figure 2. The Cartesian coordinates of the velocity $\left[\dot{x}, \dot{y}, \dot{\theta}\right]^T$ become $[u, v, r]^T$ in the local frame, linked by the relationship:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{1}$$

The wheel-ground interaction forces are called F_{x**} and F_{y**} for each one of the four wheels in both the longitudinal **x** and the lateral **y** directions (with f and r for front and rear, and l and r for left and right). Dynamical model of this classic mechanical system can be expressed in the local frame by the following equations:

$$M(\dot{u} - rv) = F_{xrl} + F_{xrr} + F_{xfl} + F_{xfr} \tag{2}$$

$$M(\dot{v} + ru) = F_{yrl} + F_{yrr} + F_{yfl} + F_{yfr} \tag{3}$$

$$\begin{aligned} J\dot{r} = {} & -w_l F_{xrl} - l_r F_{yrl} + w_r F_{xrr} - l_r F_{yrr} \\ & - w_l F_{xfl} + l_f F_{yfl} + w_r F_{xfr} + l_f F_{yfr} \end{aligned} \tag{4}$$

$$\begin{array}{lcl} J_w \dot{\omega}_{fl} = \tau_{fl} - RF_{xfl} & ; & J_w \dot{\omega}_{fr} = \tau_{fr} - RF_{xfr} \quad ; \\ J_w \dot{\omega}_{rl} = \tau_{rl} - RF_{xrl} & ; & J_w \dot{\omega}_{rr} = \tau_{rr} - RF_{xrr} \end{array} \tag{5}$$

with M the mass of the vehicle, R the wheel radius, J the vehicle inertia on **z** axis, J_w the wheel inertia, T_{**} the wheel torque, w_l and w_r the left and right width and l_f and l_r the front and rear length.

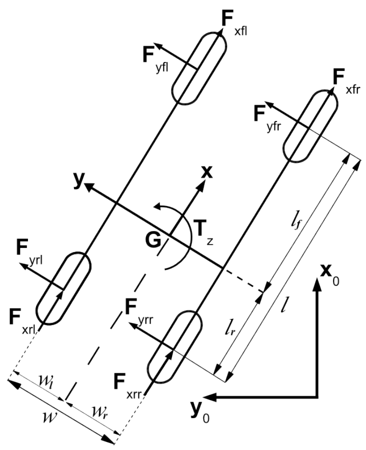


Figure 2. Vehicle dynamics

3 Control Design

Here, we use the equations of the dynamic parameters $\dot{u}$ and $\dot{r}$ for a decoupling design procedure, to control the longitudinal velocity and the yaw of the vehicle, by considering two additive torque inputs τ_u and τ_θ, defined as:

$$\tau_u = \tau_{rr} = \tau_{fr} = \tau_{fl} = \tau_{rl} \quad ; \quad \frac{\tau_\theta}{2} = \tau_{rr} = \tau_{fr} = -\tau_{fl} = -\tau_{rl} \tag{6}$$

3.1 Control of the yaw θ

Introducing the input τ_θ, equations (4) and (5) give us:

$$\dot{r} = \lambda \tau_\theta + \mathbf{K}^\theta_{\dot{\omega}} \dot{\boldsymbol{\omega}} + \mathbf{K}^\theta_{F_y} \mathbf{F}_y \tag{7}$$

with $\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dot{\omega}_{fl} & \dot{\omega}_{fr} & \dot{\omega}_{rl} & \dot{\omega}_{rr} \end{bmatrix}^T$; $\mathbf{F}_y = \begin{bmatrix} F_{yfl} & F_{yfr} & F_{yrl} & F_{yrr} \end{bmatrix}^T$; $\mathbf{K}^\theta_{\dot{\omega}} = \frac{-J_\omega}{JR} \begin{bmatrix} -w_l & w_r & -w_l & w_r \end{bmatrix}^T$; $\mathbf{K}^\theta_{F_y} = \begin{bmatrix} l_f & l_f & -l_r & -l_r \end{bmatrix}^T$; $\lambda = \frac{w_r + w_l}{JR}$.

Considering c_θ the control law and $n(\theta, r, \dot{r})$ the function of uncertainties about θ, r and $\dot{r}$ in the dynamic equations, we have the following relationship:

$$\dot{r} = c_\theta - n(\theta, r, \dot{r}) \tag{8}$$

We define the yaw control law as: $c_\theta = \dot{r}_d + K^\theta_p \varepsilon_\theta + K^\theta_d \dot{\varepsilon}_\theta + \sigma_\theta$, with $\dot{r}_d$ an anticipative term, $\varepsilon_\theta = \theta_d - \theta$ the yaw error, K^θ_p and K^θ_d two positive constants that permit to define the settling time and the overshoot of the closed-loop system, and σ_θ the sliding mode control law.

We define the error state vector $\mathbf{x} = \begin{pmatrix} \varepsilon_\theta \\ \dot{\varepsilon}_\theta \end{pmatrix}$. So, we have the state equation:

$$\dot{\mathbf{x}} = A\mathbf{x} + B(n - \sigma_\theta) \tag{9}$$

with: $A = \begin{pmatrix} 0 & 1 \\ -K^\theta_p & -K^\theta_d \end{pmatrix}$; $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

To guarantee the stability of this closed-loop system, the problem of tracking the desired raw θ_d can be solved by using the Lyapunov candidat function $V = \mathbf{x}^T P \mathbf{x}$, with P a defined positive symetric matrice. We define the sliding surface $s = B^T P \mathbf{x}$. Then, the robust sliding mode controler σ_θ is defined as $\sigma_\theta = \mu \frac{s}{\|s\|}$, with μ a positive scalar large enough to allow the stability of the controller. Based on the Lyapounov theorem (Sastry, 1999), it is proved that this controller is globally exponentially stable.

3.2 Control of the longitudinal velocity u

Introducing the input τ_u, equations (2) and (5) give us:

$$\dot{u} = \gamma \tau_u + K^u_{\dot{\omega}} \dot{\omega} + rv \tag{10}$$

with: $\gamma = \frac{4}{RM}$; $\dot{\omega} = \dot{\omega}_{fl} + \dot{\omega}_{fr} + \dot{\omega}_{rl} + \dot{\omega}_{rr}$; $K^u_{\dot{\omega}} = \frac{-J_\omega}{RM}$.

As previously, c_u is the control law and $m(u, \dot{u})$ the function of uncertainties about u and $\dot{u}$ in the dynamic equations, and we have the following relationship:

$$\dot{u} = c_u - m(u, \dot{u})$$

The longitudinal velocity control law is: $c_u = \dot{u}_d + K^u_p \varepsilon_u + \sigma_u$, with $\dot{u}_d$ an anticipative term, $\varepsilon_u = u_d - u$ the velocity error, K^u_p a positive constant that permits to define the settling time of the closed-loop system, and σ_u the sliding mode control law.
Using the Lyapunov candidat function $V = \frac{1}{2}\varepsilon_u^2$, it can be immediately verified that the stability of the system is guaranteed by the choice of the sliding mode control law $\sigma_u = \rho \frac{\varepsilon_u}{\|\varepsilon_u\|}$, with ρ a positive scalar, large enough.

3.3 Expression of the global control

In practice, uncertainties about the dynamic of the system to control have for consequence an unknown about the real sliding surface $s = 0$. As a consequence $s \neq 0$ and the sliding control law σ, which has a behavior similar to a sign function, induces oscillations by trying to reach the sliding surface $s = 0$ with a time null in theory. These high frequency oscillations, called chattering, can be reduced by adding a parameter with a small value β in the denominator.
Finally, the following torques are applied to each one of the four wheels:

$$\tau_{fl} = \tau_{rl} = \tau_u - \tfrac{\tau_\theta}{2} \quad ; \quad \tau_{fr} = \tau_{rr} = \tau_u + \tfrac{\tau_\theta}{2} \tag{11}$$

with τ_u and τ_θ defined by:

$$\tau_u = \frac{1}{\gamma}\left(\dot{u}_d + K^u_p \varepsilon_u + \rho \frac{\varepsilon_u}{\|\varepsilon_u\| + \beta_u} - K^u_{\dot{\omega}} \dot{\omega} - rv\right) \tag{12}$$

$$\tau_\theta = \frac{1}{\lambda}\left(\dot{r}_d + K^\theta_p \varepsilon_\theta + K^\theta_d \dot{\varepsilon}_\theta + \mu \frac{B^T P \mathbf{x}}{\|B^T P \mathbf{x}\| + \beta_\theta} - \mathbf{K}^\theta_{\dot{\omega}} \dot{\boldsymbol{\omega}} - \mathbf{K}^\theta_{F_y} \mathbf{F}_y\right) \tag{13}$$

To estimate the value of the lateral forces $\mathbf{F}_y$, the Pacejka (2002) theory could be used by considering the slip angle. But, due to the fact that the sliding mode control is robust, we can consider that $\mathbf{F}_y$ is a perturbation to reject, and not include it in the control law. A slip angle measure being in practice not very efficient, this solution is better.

4 Simulation

The simulation is executed using Ageia PhysX (see Craighead et al. (2007)), an highly realistic 3-dimensional dynamic environment, with the parameters given in table 1. An advanced tire slip based friction model is used in this simulator. It separates the overall friction force into longitudal and lateral components. It is represented by the function depicted in figure 3, the force being in N and the composite slip, taking into account the longitudinal slip of the tire and the slip angle, without unity.
We use here the following parameters:
Coordinates of the Extremum point A: $(1.0; 0.02)$;
Coordinates of the point B, beginning of the Asymptote: $(2.0; 0.01)$;
Longitudinal stiffnessFactor = 10000.0;
Lateral stiffnessFactor = 10000.0.
The stiffness factor is the base amount of “grip” the tire has in the specified direction.

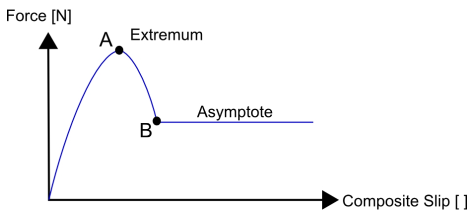


Figure 3. Friction Model

The controller parameters are chosen as: $K_p^u = 400.00s^{-1}$, $K_p^\theta = 700.00$, $K_d^\theta = 10.00$, $\beta_u = 0.10ms^{-1}$, $\beta_\theta = 0.10$, $a = 10.00$ and $b = 1.00$ (a and b are two positive constants defining the matrice Q, solution of the Lyapunov equation). The value of the torques applied in the axis of the wheels are figured with the control law designed in section 3. The simulation consists of following a path with a 90°curve. To allow the vehicle joins the path, the desired yaw angle θ_d is modified as: $\tilde{\theta}_d = \theta_d + \arctan(K_d d)$, with K_d a positive gain and d the distance to the path. Three simulations are successively done: one with only a proportionnal controler; one with $\rho = 10.0ms^{-2}$ and $\mu = 1000.00$; and finally one with $\rho = 10.0ms^{-2}$ and $\mu = 400.00$.

Table 1. Robot Parameters

Description	Symbol	Value
Length of the vehicle	l	$1.34m$
Width of the vehicle	w	$0.81m$
Height of the vehicle	h	$0.75m$
Mass of the vehicle	M	$140Kg$
Inertia of the vehicle	J	$172Kg \cdot m^2$
Radius of the wheels	R	$0.234m$
Mass of the wheels	M_w	$3Kg$
Inertia of the wheels	J_w	$0.351Kg \cdot m^2$

The displacement of the car is displayed in figure 4 and the evolution of ε_θ and ε_u is displayed in figure 5. With a proportionnal controler, the vehicle oscillates around the desired path because of the sliding phenomoenon in the wheel-soil interaction, not taken into account in the dynamic model. Adding the sliding mode control law with $\mu = 1000.00$, the path is well followed, but figure 5 shows the chattering phenomenon in the yaw error plot, which is important and can be also seen in the figure 4 at the $21.9m$ abscissa. To reduce steady state error, we have increased the value of ρ, which increases the value of the robust control input term. But, increasing ρ, the chattering phenomenon increases and the process could present non acceptable vibrations. The best behavior with a good following of the path and without chattering is plotted for $\rho = 400$. The speed error absolute value always less than $0.5ms^{-1}$ reminds quite acceptable. Notice that this controller is quite robust because the friction is not constant and some phenomena (like the elasticity of the tire for example) are not taken into account.

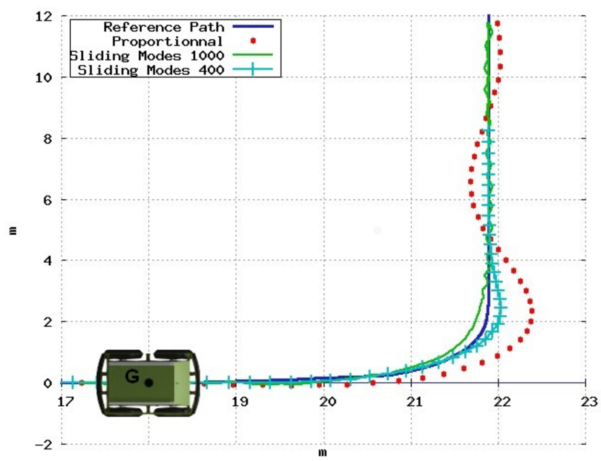
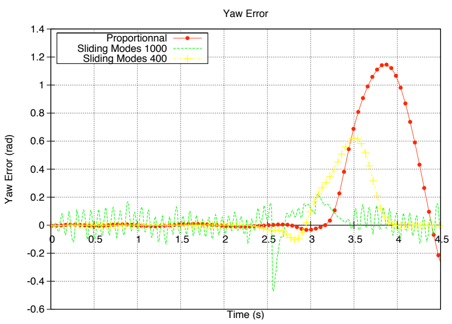


Figure 4. Robot Position



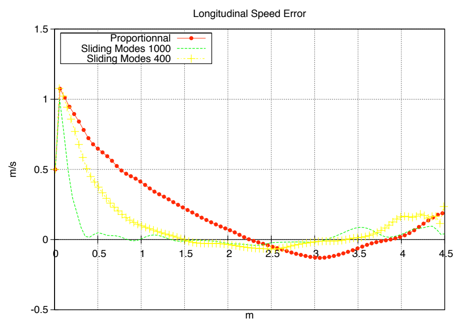


Figure 5. Yaw Error and Longitudinal Speed Error

5 Conclusion and future works

A sliding mode controller was designed and implemented on the simulated RobuROC 4 robot. The simulations performed with an accurate physical engine have shown the robustness of the control law even without any knowledge about the forces in the wheel-soil interaction. Next, we will experiment this controller in real conditions. Furthermore, it could be tested in an unstructured environment to evaluate the limits of the controler robustness. In this paper, we have not studied the problem of rollover. So, if necessary, we could use a 3D dynamic model and consider the control of the roll parameter.

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