Accurate and stable mobile robot path tracking algorithm: An integrated solution for off-road and high speed context

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Abstract—This paper is focused on the problem of accurate and reliable path tracking control of a 4-wheels car-like mobile robot moving off-road at high speed. Dynamic and extended kinematic models that take into account the effects of wheel skidding are presented. Based on the extended kinematic model, an adaptive and predictive controller for the path tracking is derived. This control law is combined to a stabilization algorithm of yaw motion, based on dynamic model and the modulation of driven wheel forces. The overall control architecture is experimentally evaluated on a slipping terrain. Results demonstrate enhanced performance as the robot succeed in following the path at high speed, accurately and without loss of control.

I. INTRODUCTION

As the autonomous navigation in off-road conditions appears as a promising solution [5] with respect to social needs in many areas (such as surveillance, rescue or agriculture [12], etc.), the research in off-road mobile robotics has to propose devices fitting users expectations. In particular, in order to be actually usable, the proposed robots have to be accurate, reliable and move at relatively important speeds. This still constitutes an open issue since natural grounds are irregular and offer low grip conditions, moreover variable (see [1]). When using basic mobile robots control law, such as proposed in [11] or in [3], these specificities indeed generate at least important perturbations (decreasing accuracy) up to a total loss of stability (spun around). Furthermore, such phenomenon are emphasized at high speed because of the unavoidable settling time and delays of actuators.

With this aim, some approaches have been developed to address instability or lack of accuracy due to low grip conditions. A first approach lies in the definition of a stability domain (velocity/steering angle) considering known grip conditions (such as proposed by [16] or in [13]). Mainly dedicated to path planning, such algorithms does not account for on-line grip condition variation in motion control. In order to compensate skidding effects in real time, an alternative consists in considering sliding as a perturbation to be rejected by robust control (see for instance [19] or [4]). If it permits to obtain a good accuracy at low speed or in a structured environment context, the settling time and delays of low level do not permit an efficient off-road control at high speed, where sliding variables can reach important values. Another way to address sliding is to control robot dynamics to make it tend to the theoretical behavior under the rolling without sliding conditions. Some work based on the wheel velocity repartition are then proposed, such as in [10], permitting to limit the effect of skidding. Nevertheless, used solely, it does not permit to obtain a high accurate path tracking, as sliding is not totally compensated.

Then, solutions based on grip conditions estimation rise at promising approaches. In [8], the authors proposed an on-line estimation procedure of the wheel-ground slippages, based on Terra-mechanics models. The slippage conditions were included in a trajectory controller in order to improve mobility over difficult terrains [9]. Nevertheless, this approach needs an accurate estimation of vehicle motion to feed tire model, which is not always practicable. Simplified models, including sideslip angles as additive variables of a kinematic representation, has been proposed and classified in [17]. Adaptive and predictive algorithms (see [2]), based on such modeling and coupled with an on-line estimation of sliding have then shown significant results from path tracking accuracy point of view. They indeed permit to estimate and compensate for perturbations whatever the changing conditions and the geometry of terrain at relatively limited speed. If the last results shown in [6], demonstrate the capability of an accurate control at high speed (compatible with low level delay), such approaches assume sliding effects are low enough to preserve the system controllability. As a result, if sliding is very large (occurring quickly at important speed levels) robot can spin around.

As a result, it appears interesting to merge solutions allowing to first reduce sliding effects on robot behavior, and then estimate and compensate remaining sliding into motion control. This paper then proposes to gather on one hand stabilization algorithm and, on the other hand, adaptive and predictive algorithm, in order to ensure a high accurate path tracking control algorithm for off-road mobile robots acting at high speed. Based on previous developments, the algorithm presented in this paper first takes part of velocity repartition on each wheel to stabilize the robot dynamics (avoiding swing around situation) and limit sliding influence. Then, an advanced path tracking control law for steering angle is derived to compensate for residual effects of sliding and anticipate low level delays. It then results a stable and accurate positioning of the mobile robot with respect to a
desired trajectory at high speed, whatever the grip condition and terrain irregularities.

The paper is organized as follows: in a first part the modeling of a mobile robot (including the reconstruction of unmeasured variables) for both part of algorithm is defined. Based on these models, a control part describing the path tracking and stabilization algorithm acting in parallel, is presented. Finally, the complementarity of developments and the relevance of the global algorithm are investigated through full scale experiments at high speed (up to 8m/s) on natural and irregular ground.

II. OFF-ROAD MOBILE ROBOT MODELING

A. Four wheels mobile robot model

First of all a complete dynamical model of mobile robot can be considered such as depicted on fig. 1. It allows to access to relationship between forces and acceleration.

In particular the four longitudinal and lateral tire forces, denoted respectively by $F_{x^*}$ and $F_{y^*}$ (with f and r for front and rear, and l and r for left and right) can be related to the yaw acceleration ($\ddot{\theta}$) thanks to the yaw torque (denoted in the following $T_\theta$).

$$\ddot{\theta} = f(F_{x^*}, F_{y^*}, \delta_l, \delta_r)$$ (1)

where $\delta_l$ and $\delta_r$ denoting respectively left and right steering angle, related to the equivalent steering angle $\delta_F$ by the robot wheelbase $L$ and the half width $W$.

The equation 1, detailed in [10], permits to analyze the effect of each forces on yaw acceleration. If the longitudinal forces $F_{x^*}$ can be controlled directly by the wheel velocity actuators, lateral forces, acting also on the yaw rate, rely mainly on steering angle, robot velocity and grip conditions. As a result, this dynamical approach appears to be suitable for yaw rate modulation via wheel velocity to reduce effects of sliding, while the steering angle actuation is investigate thanks to another level of modeling.

B. Extended bicycle kinematic model

1) Model Description: A path tracking control based on a complete model such as depicted on fig. 1 requires the knowledge of numerous parameters (hardly measurable and variable off-road). As a consequence, the design of control laws for robot motion has to be based on lighter model. Nevertheless, the kinematic model classically used in path tracking applications basically relies on the rolling without sliding assumption, which is not applicable off-road. The direct use of such control laws indeed leads to large tracking errors, due to neglected dynamics (mainly low grip conditions, actuator delays).

Consequently, an alternative model (so called "extended kinematic model") is considered in that paper, preserving a kinematic representation. As detailed in [2], it consists in adding a limited number of variables representative of low grip conditions into a pure kinematic model. As depicted in Figure 2, the two sideslip angles $\beta_F$ and $\beta_R$ (denoting the difference between tire direction and actual speed vector orientation) have been introduced into a bicycle representation of the mobile robot as in [15]. Notations, depicted on Figure 2, are listed below.

- $O$ is the center of the rear axle and constitutes the point to be controlled.
- $\beta = \theta - \theta_\Gamma$ is the vehicle angular deviation with respect to $\Gamma$.
- $v$ is the vehicle linear velocity at point $O$, assumed to be strictly positive.
- $\beta_F$ and $\beta_R$ are the front and rear side slip angles.
- $M$ is the point on the path $\Gamma$ to be followed, which is the closest to $O$. $M$ is assumed to be unique.
- $c(s)$ is the curvature of the path $\Gamma$ at point $M$, pending on $s$ the curvilinear absissa.
- $y$ is the vehicle lateral deviation at point $O$ with respect to $\Gamma$.

Except the two sideslip angles $\beta_F$ and $\beta_R$, all the variables described are supposed to be measured or known by a preliminary calibration. Thanks to this representation framework, the evolution of the vehicle state with respect to the path $\Gamma$ to be followed can be described by the set of equations (2) (see [2] for more details).
A. Adaptive and predictive control

The extended kinematic model (2), coupled with the observer described on Figure 3, allows an accurate description

\[
\begin{align*}
\dot{s} &= v \frac{\cos(\theta + \beta_R)}{1 - c(s) y} \\
\dot{y} &= v \sin(\theta + \beta_R) \\
\dot{\theta} &= v \frac{\cos(\beta_R) \lambda_1 - \lambda_2}{L}
\end{align*}
\]

with: 

\[
\lambda_1 = \frac{\tan(\delta_F + \beta) - \tan(\beta_R)}{L} \quad \lambda_2 = \frac{c(s) \cos(\beta_R) \lambda_1}{1 - c(s) y}
\]

The existence of this model is guaranteed since \( y \neq \frac{1}{c(s)} \)

(i.e. the point \( O \) is supposed to be never at the center of the reference path curvature). It can also be checked that classical kinematic model defined in [14] can be found by setting sideslip angles to zero. The proposed structure is then consistent with classical point of view. As a consequence, as soon as sideslip angles are correctly estimated, the properties of such kinematic structures as well as results can be applied.

B. Modulation of wheel velocity for yaw rate regulation

Due to the presence of slippage in the wheel-ground contact, especially at high speed, estimated sideslip angles can reach high values. Such sliding levels may not be compensated properly by path tracking algorithm and lead to swing around. In such case, the actual yaw rate of the robot \( \dot{\theta} \) is then widely different from the theoretical yaw rate under rolling without sliding condition \( \dot{\theta}^t \) defined as:

\[
\dot{\theta}^t = \frac{v \tan \beta}{R}
\]

PENDING on the ground slippage conditions and the dynamics configuration of the vehicle, over-steer (\( \dot{\theta} > \dot{\theta}^t \)) or under-steer (\( \dot{\theta} < \dot{\theta}^t \)) appear during turning maneuvers. In order to reduce such differences, consequently decreasing sideslip angles and finally improve efficiency of the control law (6),
an additive regulation of yaw rate is designed. On the partial dynamic model given in equation (1), the modulation of longitudinal forces produced by the wheels is considered. First, the resulting yaw moment $T_θ$ acting on the system for a given steering configuration ($δ_F$) and a given distribution of propulsion forces $F_{x+s}$ is analyzed. The influence of these propulsion forces variation on yaw motion is given by the variation of $\frac{dT_θ}{dT_{F_{x+s}}}$. Let us define the propulsion forces as a function of the force resulting from the extended kinematic controller $F_{x+s}^k$ and a force added to stabilize the yaw motion $F_{x+s}^\epsilon$.

$$F_{x+s} = F_{x+s}^k + F_{x+s}^\epsilon \quad (8)$$

Now, let us consider the error between theoretical yaw rate (without sliding) and measured one, defined as $\epsilon = \dot{\theta} - \dot{\theta}$. The goal of the stabilization algorithm is then to determine a set of propulsion forces that produce equivalent yaw moment $T_θ$ that compensates this error ($\epsilon$ see [10] for details).

$$\begin{cases}
F_{x+s}^\epsilon = \Phi(\delta, \epsilon) \\
\epsilon = \dot{\theta} - \dot{\theta} \quad (9)
\end{cases}$$

In the case where the system control inputs are the wheel velocities instead of the wheel torques, one can use wheel angular acceleration which is homogeneous to wheel torque (and consequently to longitudinal force), as it is claimed, for example, in [18]. Thus, the stabilization control law becomes:

$$\begin{cases}
\frac{d\omega_s}{dt} = K_\omega \Phi(\delta_F, \epsilon) \\
\epsilon = \dot{\theta} - \dot{\theta} \quad (10)
\end{cases}$$

and, then, the wheel velocity to be applied on one of the wheel ($\epsilon$s) at instant $k$ can be computed as follow:

$$\omega_s^k = \omega_s^{k-1} + \frac{d\omega_s}{dt} T_\epsilon$$

where $K_\omega$ is the conversion constant between propulsion force and wheel acceleration, $T_\epsilon$ is the sampling period and $\omega_s$ is the wheel velocity computed from the extended kinematic controller. The function $\Phi$ is detailed in [10] and can be summarized as follows:

$$\begin{cases}
F_s = -K \epsilon \\
\text{If } (\delta < 0) \land (\epsilon < -\epsilon_l) \text{ then } F_{x+1}^s = F_s \\
\text{If } (\delta < 0) \land (\epsilon > \epsilon_l) \text{ then } F_{x-1}^s = F_s \\
\text{If } (\delta > 0) \land (\epsilon < -\epsilon_l) \text{ then } F_{x+1}^s = F_s \\
\text{If } (\delta > 0) \land (\epsilon > \epsilon_l) \text{ then } F_{x-1}^s = F_s
\end{cases}$$

The limit $\epsilon_l$ defines the threshold of activation of this wheel velocity control (WVC ) and $K$ is a strictly positive constant.

C. Global algorithm

Both of the proposed control approaches act on different steering part of a car like mobile robot. While the adaptive and predictive law (6) is devoted to steering angle for path tracking, the regulation of yaw rate defined by (12) is applied on one wheel velocity. This latter regulation then permits to make the robot behavior closer to theoretical motion under rolling without sliding, consequently reducing the level of sliding to be accounted by the steering control law. This reduction in sliding level then improves the efficiency of the path tracking, preventing the robot from swing over situations (due to too large sideslip angles). As the incidence of WVC on sideslip angles can be accounted on-line thanks to the observer defined on the figure 3, both of control laws can be applied in parallel such as on the scheme depicted on the figure 4.

IV. EXPERIMENTAL RESULTS

A. Experimental mobile platform

The experimental platform is shown in Figure 5. It consists of an electric off-road vehicle, whose maximum reachable speed is $8 m.s^{-1}$. Designed for all-terrain mobility, it can climb slopes up to 45° and has the following properties:

| Total mass | $m = 350 kg$ |
| Wheelbase | $L = 1.2 m$ |
| Rear half-wheelbase | $l = 0.58 m$ |

**TABLE I**

EXPERIMENTAL ROBOT DYNAMIC PARAMETERS

The main exteroceptive sensor on board is a Magellan ProFlex 500 RTK-GPS receiver, which can supply an absolute position accurate to within 2 cm, at a 20Hz sampling frequency. The GPS antenna is located vertically above the center of the rear axle, so that the absolute position of point $O$ (i.e. the point to be controlled, see Figures 2) is straightforwardly obtained from the sensor. In addition, a gyrometer supplying a yaw rate measurement accurate to within 0.1°.s$^{-1}$ is fixed on the chassis, to feed both the observer algorithm, and the separated wheel velocity control (WVC ).

B. Stabilization results

The first contribution of the proposed approach lies in the trajectory tracking stabilization of mobile robots acting at high speed on natural ground. In order to point out the effect of the distributed wheel velocity control, an half turn has been first manually recorded on a wet grass ground at a speed of $1 m.s^{-1}$. This trajectory (depicted in black plain line on the Figure 6) constitutes the reference path.

This path has then firstly been followed automatically using only the front steering angle control law (6), without the differential wheel velocity control at a velocity of $8 m.s^{-1}$. The trajectory resulting from this control is reported...
in red dashed line on figure 6, showing the instability, since the mobile robot swings around during the curve. On the contrary, the same control law extended with the wheel velocity control WVC has then be used at the same speed. As it can be seen (trajectory resulting from this test is reported in blue dashed-dotted line on the same figure), the mobile robot is then able to follow entirely the reference path with a limited tracking error (below 1.5m at 8m.s$^{-1}$).

As the stabilization algorithm attempts to make the robot yaw rate converging to the ideal one (computed in non sliding case), it mechanically makes the vehicle sideslip angles decrease. The robot behavior under WVC is indeed closer from the behavior of a robot moving under good grip condition and the variations of steering control are consequently less important, preventing the robot from spin around situations.

To show the influence of the stabilization on the estimated sideslip angles, the Figure 7 compares the front estimated sideslip angles during both of the experiments (using the same conventions than on the previous figure). The swing around situation can then be identified in the case without WVC, around curvilinear abscissa 35m, since estimated sideslip angle saturates to -60° (computed limit). On the contrary, when WVC is active, the sideslip angle is significantly reduced with a minimum transitional value of 45 allowing to preserve the controllability for the path tracking algorithm. Such a result is then obtained thanks to the differential braking. The result of the speed limitation for each wheel obtained during the tests without swing around is reported in percentage on the figure 8.

At the beginning of the path following just before the curve, one can see on the figure 8 that one wheel velocity decreases of about 30 percent (the rear right wheel) to prevent oversteering for a negative yaw rate error. Then, when the vehicle is turning to the left in the positive $\theta$ direction, it understeers, so the yaw rate error becomes positive ($\varepsilon > 0$) and a velocity decrease of almost 60 percent is applied to the rear left wheel. At the end of the curve, a velocity decrease of more than 20 percent and then about 35 percent are successively applied to the front right and the rear right wheels to prevent an oversteering in the curve exit.

C. Path tracking accuracy results

If the stabilizing algorithm permits to reduce the influence of sliding, the remaining skidding effects have nevertheless to be addressed in order to preserve the path tracking accuracy. The use of WVC without integrating sliding effects by adaptive control (4) and (6) indeed leads to large error, especially at 8m.s$^{-1}$ on the previous trajectory. The lateral error then reaches more than 5m during the curve (mobile robot then stops for security reasons). In order to point out benefits of adaptive control gathered with WVC, results of tracking errors obtained at lower speed (6m.s$^{-1}$), with different configurations of the control algorithm for the path depicted on Figure 6, are proposed on the Figure 9.

On this figure, the first tracking error reported (in black plain line) is the result of a classical path tracking control
law (sliding is neglected), and without differential wheel control velocity. It can be noticed that during the curve, a large deviation (close to 4m) is recorded. As has been pointed out, the differential control of wheels allows to reduce the effect of sliding. Consequently, the deviation reported in blue dashed line, when only WVC is active (sliding is neglected in steering control), is slightly reduced, but still important (up to 2.5m) during the curve. The same remark can be achieved when using only the steering control (with sliding accounted, but WVC inactive), the error of which is reported in green dotted line. The error is considerably reduced during the curve, but large deviations are recorded at the end of the curve, because of the huge variation of sliding during the transition curve/straight line. Finally, when combining both of the algorithms (WVC active and sliding accounted), it can be noticed that tracking error (reported in magenta dashed dotted line) is significantly reduced all path long. The largest error is indeed limited to 1m (punctual overshoot at abscissa 40m), while the behavior is much more stable (in terms of oscillations).

Benefits of the complete algorithm can be tested further on more complex trajectories, even non admissible at high speed. For instance, a "double S" trajectory has been recorded manually at a quite limited speed of 1m.s$^{-1}$, still on a wet grass ground. This reference path, depicted in black line on the Figure 10, is quite difficult to follow properly at high speed because of low level delays and high speed transition of sideslip angles. It finally becomes non admissible (it is physically not achievable) at the maximal speed of 8m.s$^{-1}$.

![Figure 10. Comparison of path in double S at different speed](image)

Fig. 10. Comparison of path in double S at different speed

Nevertheless, the proposed algorithm permits to ensure the stability of the path tracking, even during the harsh conditions with a limited deviation, as it can be noticed by considering the trajectories obtained with the entire algorithm at 4, 6 and 8 m.s$^{-1}$. If the first curve is similarly followed at each of the considered speed, the fast modification of curvature sign generates unavoidable but limited overshoots at high speed, so does in the second S. Nevertheless, the stability stays ensured and mobile robot does not swing around.

To go further and point out contributions brought by all parts of the proposed algorithm, the comparison of several lateral errors resulting from different configurations of the path tracking algorithm at a speed of 4m.s$^{-1}$ (as the reference path is still admissible at this speed) is reported on figure 11.

![Figure 11. Comparison of tracking errors in double S at 4m.s$^{-1}$](image)

Fig. 11. Comparison of tracking errors in double S at 4m.s$^{-1}$

As it can be noticed, a control law based on classical model (sliding neglected) appears to be quite inaccurate during the curve. As the influence of grip conditions are not accounted, important deviations (more than 2m) are recorded during the curves (between curvilinear abscissas 25-38m and 55-65m). As expected the WVC activation (tracking error is reported in blue dashed line) permits to limit the effects of slip since it attempts to make the robot behavior close to a non sliding one. As a result the error is slightly reduced during both of the S curves with a maximal deviation limited to 1.2m. Finally, the benefit of merging WVC and adaptive and predictive algorithm (integrating an indirect estimation of sliding), then clearly appears. The error related to the result obtained with the complete algorithm (in magenta dashed-dotted line) stays very close during all the path tracking. Despite the harsh transition (linked to the reference path geometry), the bad grip conditions (wet grass soil and low tire width) and the relatively high speed, the tracking error does not exceed 0.45m. It finally show the complementarity of advanced path tracking algorithm and WVC for an accurate and stable path tracking in the considered conditions.

V. CONCLUSION AND FUTURE WORK

A robust complete controller of generic 4-wheel-steering mobile robots has been presented. This controller is suitable for high speed path tracking on uneven winding terrains. It is able to handle sliding sohs to preserve accuracy and stability of path tracking control. Results obtained with the implementation of complementary algorithms have shown their efficiency in such conditions. Further, the extension of this work to a stabilization algorithm acting simultaneously on the four wheels of the robot is currently being investigated, as well as the integration of stability with respect to rollover risk.

REFERENCES


