Large Obstacle Clearance Using Kinematic Reconfigurability for a Rover with an Active Suspension

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This paper deals with the clearance capabilities of mobile robots in rough terrain. A way of using kinematic reconfigurability is proposed to allow the crossing of obstacle that would normally be impossible by choosing a configuration that will guaranty static equilibrium. The control uses force control on the legs and try to decrease the internal forces needed to insure stability.

Keywords: Kinematic reconfigurability, Obstacle crossing, Mobile robot

1. Introduction

There is an increasing need for autonomous vehicles to navigate through extremely rough terrain. From military applications to planetary exploration, robots will have to cross more different type of terrains, including large obstacles. Researchers have worked on mechanical design\(^1,2\) to improve the clearance capabilities of autonomous vehicles, and on control algorithms\(^3-5\) to optimize friction forces. Others\(^6,7\) have worked on kinematically reconfigurable systems to improve the tipover stability of the robot. This paper proposes a way to use kinematic reconfigurability of the 4 wheels mobile robot HyLoS 2 to optimize contact forces with respect to non slippage constraints and tipover stability constraints during large obstacle crossing.

First, the quasi-static model of the system is described, the forces distribution problem under non-slippage constraint is formulated and the ability for the robot to cross an obstacle in a given configuration is analyzed. Then, a way of finding a configuration in which the robot is able to cross the obstacle which minimizes the forces needed to guaranty static equilibrium is presented. The algorithm is evaluated in simulation.
2. Quasi-static model of the HyLoS 2

The HyLoS 2 (Fig. 1 and Fig. 2) is composed of a chassis of mass $m$ sustained by four legs, each having a wheel at its end. The legs have two revolute joints along the lateral axis ($y$), allowing them to apply a force in the sagittal plane ($x, z$), a revolute joint for the wheel direction and a last one for the wheel actuation. It is assumed that the contacts between the wheels and the ground are contact points with friction and that no moment is transmitted through them.

The quasi-static model of the system is given by:

$$ Gf_c = F_d $$

where $f_c[12\times 1]$ is the vector of contact forces expressed in the global frame $\mathbb{R}_o(O, x_0, y_0, z_0)$, $F_d[6\times 1]$ is the vector containing the desired equivalent forces ($f_d$) and moment ($m_d$) on the center of mass, and $G_{6\times12}$ is the transformation matrix which give the equivalent wrench at the center of mass of the robot of the contact forces:

$$ G = \begin{bmatrix} I_{3\times3} & \cdots & I_{3\times3} \\ \tilde{p}_1 & \cdots & \tilde{p}_4 \end{bmatrix} $$

where $\tilde{p}_k$ is the skew symmetric matrix of the cross-product associated to the vector of the $k^{th}$ contact point’s position expressed in $\mathbb{R}_c(C, x_0, y_0, z_0)$ where $C$ is the center of mass of the robot.

The contact forces $f_c$ solution of (1) must satisfy the non-slippage conditions given by the Coulomb friction model and guaranty positive normal forces. These constraints are given for the $k^{th}$ contact point by:

$$ f_{n_k} > 0 $$

$$ \mu^2 f_{n_k}^2 > f_{t_k}^2 $$

where $f_{n_k}$ is the component of the contact force normal to the contact plane of the $k^{th}$ wheel, and $f_{t_k}$ is the component tangential to the contact plane for the same wheel.

As this paper is only interested in frontal obstacle crossing, the movement will be in the sagittal plane. A two-dimensional half car model in the sagittal plane is considered. It has 2 wheels in contact with the ground and it is assumed that the contact forces along the lateral axis are null. $G$ is then a $[3 \times 4]$ matrix, $f_c$ is a $[4 \times 1]$ vector and $F_d$ is a $[3 \times 1]$ vector. The contact constraints are then given by:

$$ f_{n_k} > 0 $$

$$ \mu f_{n_k} - f_{t_k} > 0 $$

$$ -\mu f_{n_k} - f_{t_k} < 0 $$
3. Contact forces optimization

The contact forces are decomposed in 2 terms: \( \mathbf{f}_c = \mathbf{f}_0 + \mathbf{f}_i \). The first term \( \mathbf{f}_0 \) is the solution of (1) which give the two contact forces parallel to the resultant force \( \mathbf{f}_d \). As long as the two contact forces have the same direction \( \mathbf{f}_{01} = b \mathbf{f}_{02} \) with \( b > 0 \), this solution is the one minimizing the sum of contact forces’ norms. The second term \( \mathbf{f}_i \) corresponds to the internal forces, solution of the constraint \( \mathbf{G} \mathbf{f}_i = 0 \).

The problem now becomes: find the internal forces which respect the contact constraints (4).

The internal forces for a two contact points system are described by two opposite forces whose norms are equal and of direction along the line that joins the contact points:\(^8\)

\[
\begin{align*}
\mathbf{f}_{i1} &= f_i \mathbf{u}_{i1} \\
\mathbf{f}_{i2} &= f_i \mathbf{u}_{i2} 
\end{align*}
\]  

(5)

where \( \mathbf{f}_{i1} \) and \( \mathbf{f}_{i2} \) are the internal forces applied on the rear wheel (index 1) and front wheel (index 2), \( f_i \) is the amplitude of the contact forces. \( \mathbf{u}_{i1} \) and \( \mathbf{u}_{i2} \) are the vectors giving the direction of the internal forces \( \mathbf{u}_{i1} = -\mathbf{u}_{i2} \).

\( \mathbf{u}_{ik} \) is expressed in the contact frame as:

\[
\mathbf{u}_{ik} = \begin{bmatrix} u_{in_k} \\ u_{it_k} \end{bmatrix} 
\]

(6)

where \( u_{in_k} \) is the normal component of the internal force and \( u_{it_k} \) is its tangential component.
The contact force applied at the $k^{th}$ contact point and expressed in the contact frame is:

$$f_c = \begin{bmatrix} f_{n_k} \\ f_{t_k} \end{bmatrix}$$ (7)

then the relation $f_{c_k} = f_{0_k} + f_{i_k}$ gives:

$$f_{n_k} = f_{0n_k} + f_{i_n k}$$
$$f_{t_k} = f_{0t_k} + f_{i_t k}$$ (8)

The constraints (4) can now be expressed as bounds on the internal forces norm:

$$f_i u_{in_k} > -f_{0n_k}$$
$$f_i (\mu u_{in_k} - u_{it_k}) > -(f_{0n_k} - f_{0t_k})$$
$$f_i (\mu u_{in_k} + u_{it_k}) < -(f_{0n_k} + f_{0t_k})$$ (9)

Depending on the terms of $u_{ik}$, each of these six constraints can lead to a lower or an upper bound on $f_i$. These bounds are called $b_j$ with $j \in [1, 6]$. Let $U \{ j / f_i < b_j \}$ be the set of index giving upper bounds and $L \{ j / f_i > b_j \}$ the set of index giving lower bounds. The internal force's norm must then be within the range $[b_{low}, b_{up}]$ where $b_{low} = \max_{j \in L}(b_j)$ and $b_{up} = \min_{j \in U}(b_j)$. If the upper bound is lower than the lower bound, it is impossible for the robot to satisfy all the constraints, it can not be in static equilibrium with its posture.

As energy consumption is critical for rovers, the internal forces which have the lowest possible norm given the constraints are chosen.

4. Influence of the posture on the equilibrium

Let a posture be defined by $X$, $Z$, and $\Phi$. $X$ and $Z$ are the horizontal and vertical position of C expressed in $\mathbb{R}^d(D, x_0, y_0, z_0)$ where $D$ is the center of the segment joining the two contact points. $\Phi$ is the orientation of the chassis around $y_0$. In the case where $f_d = mg$ is along the vertical axis, each bounds of the internal forces can be expressed as function of $X$ only such as:

$$b_j = a_j X + c_j$$ (10)

So, according to (10), the range $[b_{low}, b_{up}]$ can be changed by modifying the posture of the robot.

The following presents an analysis of the bounds $b_{low}$ and $b_{up}$ with 2 different postures for a step crossing. The angle between the ground and the horizontal is 0 on the rear wheel and $\pi/2$ on the front wheel. The coefficient
of friction is $\mu = 0.6$. The bounds are plotted as function of $\theta_a$, the angle between the line joining the contact points and the horizontal. The bounds are plotted on Fig. 3 for a nominal posture and on Fig. 4 for a posture where the chassis is moved backward.

The results show that before $\theta_a$ reach 0.32, the robot can not achieve static equilibrium with a nominal posture but he can if his chassis is moved backward. On the other hand, the robot with the modified posture will fall for $\theta_a$ greater than 0.52 when he would have remained stable with the nominal posture.

The conclusion of these results is that it is not always possible to find a fixed posture allowing the robot to cross a given obstacle. The robot must adapt his posture during the motion across the obstacle.

![Fig. 3. upper and lower bound for a step crossing with a nominal posture](image1)

![Fig. 4. with the chassis moved backward](image2)

5. Control law with posture optimization

The goal of the control law presented here is to find a posture that guaranty static equilibrium under the assumption that only the gravity is applied on the robot. As for the internal forces, the chosen posture must minimize the contact forces. The search for this posture is illustrated on Fig. 5. As the bounds are affine functions of $X$, the postures where the bounds are at a local minimum are the intersection points between the bounds or between the bounds and zero. To find the horizontal position of the center of mass $X_d$ which minimize the contact forces, we must first filter out the infeasible points, ie the points where $b_{low} > b_{sup}$, and choose the posture with the minimum needed force among the remaining points.
The quasi-static model is solved as described before, $f_d$ is set to compensate gravity and to seek the desired posture:

$$f_d = mg + ma$$

where $m$ is the mass of the robot, $g$ is the gravity acceleration and $a$ is the desired acceleration of the center of mass allowing the posture correction as:

$$a = [K_p(X - X_d) + K_c \dot{X}]x_0$$

The control torques are then calculated using the Jacobian matrix:

$$\tau = J^T(f_c + f_p)$$

The control scheme is showed in Fig. 6.
6. Simulation results

The control law exposed before is evaluated in simulation using the physics engine Bullet. Two situations are simulated. For each of them, a comparison of the results obtained with and without the posture optimization is made. Without it, the robot’s posture is fixed.

The first situation concerns a step crossing (contact angle of $\pi/2$ on the front wheel). The ratio between the tangential and the normal forces needed to achieve static equilibrium are shown on Fig. 7. If the robot doesn’t use the posture optimization, this ratio is greater than the friction coefficient of 0.8, the robot can not cross the obstacle because the wheels are slipping. The horizontal position $X$ of the robot with the posture optimization is plotted on Fig. 9.

The second situation is the crossing of an obstacle with a 1.2 radian angle between the ground and the horizontal on the front wheel. The goal of this simulation is to show the effect of kinematic reconfiguration on an obstacle that can be crossed without it. As showed on Fig. 8, the internal forces needed to achieve static equilibrium are lower with the posture optimization than without.

7. Discussion and conclusion

This paper presented a way of using reconfigurability to enhance the crossing capability of the robot. The first result is that this control law allows the crossing of difficult obstacles such as steps higher than the wheel’s diameter. Furthermore, the reconfiguration reduces the internal forces needed to cross an obstacle. The simulated example shows that a 20% reduction of these forces can be achieved.

An other way of using kinematic reconfigurability could be to increase the robustness of the stability during the obstacle crossing. Further work will concern the implementation of the control law on the physical prototype.

References

Fig. 7. Ratio between the tangential force and the normal force on the front wheel for a step crossing.

Fig. 8. Internal force’s norm during an obstacle crossing.

Fig. 9. Horizontal position of the center of mass of the robot (X) during a step crossing.

Fig. 10. Horizontal position X during an obstacle crossing.


