We propose new nonlinear PI/PID controllers, with the constraints of control saturation, for first-order and second-order SISO systems. These controllers offer:

- the performance of Time-Optimal Controls,
- the robustness of classical PI/PID Controls,
- limited integrator wind-up effects.

**Continuous feedback approximation**

We consider System (1) and approximate the TOC (2) by the continuous Time Sub-Optimal Control (TSOC)

\[
\begin{align*}
\dot{x}(t) &= u(t) = -k_p x - k_s \text{sat}_m^s(\dot{x}(t)) - \text{sat}_l^l(k_c \dot{x}(t)) \\
\end{align*}
\]

where the \( \text{sign}_m^s(\cdot) \) function is replaced by the \( \text{sat}_m^s(\cdot) \) function, and \( a(x, \dot{x}) \) is replaced by the continuous function \( a(x, \dot{x}) \) defined by

\[
\begin{align*}
a(x, \dot{x}) := \frac{M - m}{2} + \frac{M + m}{2} \text{sat}_l^l\left(\frac{\dot{x}}{\varepsilon}\right)
\end{align*}
\]

Positive design parameters : \( k_p, k_s, \varepsilon, l \).

**Remarks on the TSOC (3)**:

- The linear approximation of (3) is the classical PD controller \( u(x, \dot{x}) = -k_p x - k_s \dot{x} \).
- Thus, it locally inherits the properties of this feedback control.
- The TSOC (3) approximates the TOC (2) when the “error” \( (x, \dot{x}) \) is initially large. The approximations are all the better than \( k_p \) is large and \( \varepsilon \) is small.
- The TSOC (3) is a global asymptotical stabilizer of the origin if \( \varepsilon < \min(M, -m)/k_p \).

**Integral action component**

We extent the TSOC (3) so as to deal with the following perturbed SISO system

\[
\begin{align*}
\dot{x} &= u + c, \quad m \leq u \leq M, \quad |c| < \min(M, -m) \\
\end{align*}
\]

The following “bounded” conditional integrator is proposed in order to compensate for the unknown constant perturbation input \( c \), and to limit integrator wind-up effects

\[
\begin{align*}
\dot{z} &= -k_{pz} \dot{z} + \text{sat}_l^l\left(\frac{m_p}{\max}\left(-z + \text{sat}_l^l(z + x)\right)\right)
\end{align*}
\]

with \( k_{pz}, m_p, \max, \delta \) positive numbers, \( |z(0)| < \delta max/(2k_{pz}), \ |\dot{z}(0)| < \max/(2k_{pz}) \).

\( z(t), \dot{z}(t), |\dot{z}(t)| \) are uniformly bounded by \( \delta_z + \max/(2k_{pz}), \max/(2k_{pz}) \).

The proposed Time Sub-Optimal PID Control extends the TSOC (3) as follows

\[
\begin{align*}
u &= \text{sat}_m^s\left(\frac{-k_p}{2a(x, \dot{x})} - \text{sat}_l^l(k_c \dot{x}) - \dot{z}(\dot{x}, z, \dot{z})\right) \\
\end{align*}
\]

where \( \dot{x} := x \dot{z} := \dot{x} \dot{z}, \ M := M - \max, \ m := m + \max, \ a(x, \dot{x}) \) is replaced by (4) with \( M \) and \( m \) replaced by \( M \) and \( m \) respectively.

Control insights : In the case where \( c = 0 \), the augmented control system writes

\[
\begin{align*}
\dot{\tilde{x}} &= \tilde{u} := u + \dot{z}(\dot{x}, z, \dot{z}) \leq u \leq M
\end{align*}
\]