A Sufficient Condition for Force-Closure
Grasps Synthesis of 3D Objects

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Problem Statement

Generating a grasp requires information about:
- Contact Type
- Contact Number
- Local Object Surface

When Does a Grasp Have Force-Closure?

According to the definition of Salisbury and Roth, a grasp has force-closure if and only if any external wrench can be balanced by the wrenches at the fingertips — when the primitive contact wrenches resulted by contact forces at the contact points positively span the entire 6-dimensional wrench space.

Can 6D Contact Wrenches Be Represented in a 3D Space?

- A primitive contact wrench is defined by: \( w = (f \times p) \).
- Using Plücker coordinates, \( w \) can also be seen as a representation of the line of action \( L \) of the force \( f \) applied at the point \( p \).

Key Ideas

- All lines through one point are of rank 3.
- When all lines meet one line, they are of rank 5.
- Wrenches associated to \( n \) non-aligned contact points are of rank 6 and thus form a basis of the 6D wrench space.

The Sufficient Condition

Assume that:
- the grasp of \( n \) \( n \)-finger non-aligned fingers is not force-closure.
- \( \{b_i\}_{i=1}^n \) is a \( n \)-dimensional basis associated to their corresponding contact wrenches.

Sufficient condition for a \( n \)-finger force-closure grasp is that there exists a contact wrench \( \gamma \) such that:

\[
\gamma = \sum_{i=1}^{n} \beta_i b_i, \quad \beta_i < 0 \Rightarrow \gamma = B\beta \Rightarrow \beta = B^{-1}\gamma
\]

where:
- \( B = [b_1, b_2, ..., b_n] \) is a \( 6 \times n \) matrix.
- \( \beta = [\beta_1, \beta_2, ..., \beta_n] \) is a \( 6 \times 1 \) strictly negative vector.

Results and Conclusions

Tests are accomplished on a sphere model, represented by its 762 vertices and its respective normal directions. Two experiments are performed:

Completeness test:
- Depends on the quality of the first \( n-1 \) generated fingers locations. Their quality should be above a threshold.
- In the worst case, when the \( n-1 \) fingers are generated randomly, our method finds 18.7% of the force-closure grasps.

Rapidity test:
- Depends on the quality of the first \( n-1 \) generated fingers locations.
- In the worst case, our force-closure computation time is four times faster than the convex-hull one.

<table>
<thead>
<tr>
<th>Number of Solutions (s)</th>
<th>Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold</td>
<td>Complete</td>
</tr>
<tr>
<td>Th0</td>
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</tr>
<tr>
<td>Th1</td>
<td>349</td>
</tr>
<tr>
<td>Th2</td>
<td>386</td>
</tr>
<tr>
<td>Th3</td>
<td>392</td>
</tr>
<tr>
<td>Th4</td>
<td>419</td>
</tr>
</tbody>
</table>

Tab. 1 - Completeness results for different thresholds: \( 0 = Th0 < Th1 < Th2 < Th3 < Th4 \).

<table>
<thead>
<tr>
<th>Time (ms)</th>
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<tbody>
<tr>
<td>Threshold</td>
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<tr>
<td>Th0</td>
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<td>Th1</td>
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<td>Th2</td>
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<td>Th3</td>
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<td>Th4</td>
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Tab. 2 - Force-Closure grasp computation time

Our aim is to compute as fast as possible \( n \)-finger force-closure grasps for a given 3D object.

The proposed force-closure sufficient condition is not necessary. Our method sacrifices completeness in favor of fast computation.

Fast computation is due to reducing the force-closure test to an inverse matrix calculation.

In the worst case, our approach is four times faster than the convex-hull method.